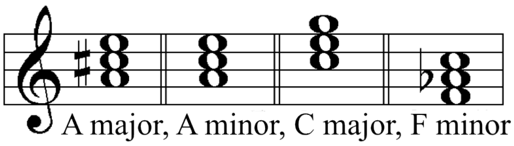
I. Harmonics

1. Consonance

a) Harmony of the Basic Consonance

Tonal music is music on a harmonic basis. The key to its aesthetics lies in the analysis of the major and minor triads, the basic sound forms of the tonal music. In the progress of the analysis, it will become apparent how any harmony, rhythm, and melody of the tonal music is built upon the harmony of these two sound figures.

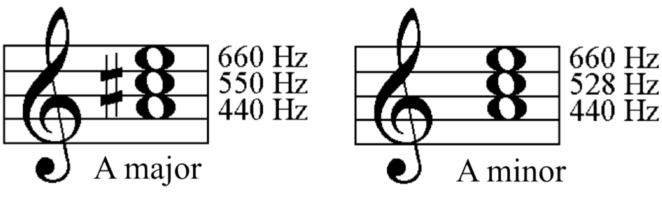


Major and minor triads

Major and minor triads are triads whose tones harmonise with each other. What characterises these triads are the *relations* between the three tones called the root, third, and fifth. Only *because of and within these relations*, musical tones are defined as a root, third, and fifth. It is therefore quite appropriate to name the *tones* third and fifth after their *relation* to the root, which in turn would not be a root tone without these relations.

The relation between the tones third and fifth is again a third. Each major or minor triad contains in its basic form a major and a minor third, which complete each other to form a fifth. The major triad has the major third in the bottom part, the minor triad has it on top. Both sound forms have something in common: a specific harmony, which is called consonance. The essence of this harmony shall now be explained.

The harmony of the major and minor triads is obviously related to the frequency ratios that are characteristic of these sound forms:



Tone frequencies of two triads

The frequency ratios of these triads are:

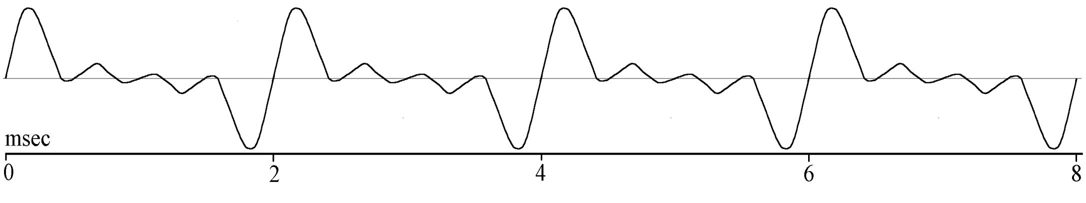
Fifth:	660 Hz : 440 Hz		= **3:2**
Major third:	550 Hz : 440 Hz	= 660 Hz : 528 Hz	= **5:4**
Minor third:	660 Hz : 550 Hz	= 528 Hz : 440 Hz	= **6:5**

The mathematical ratios between the frequencies of harmonising tones are of interest in instrument making, where these proportions have to be considered with regard to the dimensions of resonating cavities or strings. An organ pipe, for example, has to be the longer the lower its tone and the smaller therefore the related tone frequency. The frets on a guitar neck must be positioned in such a way that, for example, the tones of a major triad can be played one after the other on a single string, which presupposes that the vibrating string parts have the frequency ratio 4:5:6. Since the lengths of vibrating string parts are inversely proportional to the number of vibrations per second, the numerical ratios that are decisive for the harmonising tones have a clear appearance. It is therefore not surprising that the proportion 2:3 as a length ratio of string parts was known long before the knowledge of tone frequencies, namely, already in ancient times.

However, the quantitative relations observable in connection with major and minor triads (the proportions 3:2, 5:4, and 6:5) merely indicate the *external* relations between the tone frequencies. The question of the harmonic character of these sound relations is thereby in no way answered. Harmony, as a characterisation of what is perceived during the sounding of major and minor triads, means, namely, an *inner* relation of the tones respectively sounding together: a relation in which the musical tones go well together.

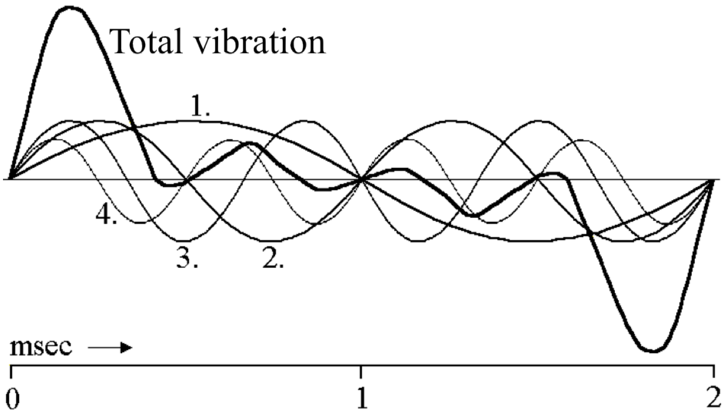
The basis for the fact that contents of perception go well together – and, by the way, that is what any aesthetics is all about – lies always in the properties of these things. In the case of major and minor triads, it is obviously the tones themselves that have something about themselves that makes them go well together. This property of the musical tones has therefore to be examined more closely.

A tone with a frequency of 500 Hz oscillates by definition 500 times per second, that is, once every two milliseconds. On an oscillograph, these oscillations are displayed, for example, as follows:



Four oscillations of a tone of 500 Hz

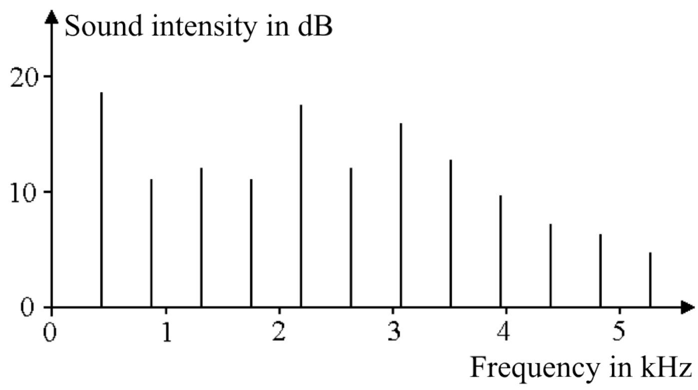
As the physicist and mathematician Jean Baptiste Fourier has generally demonstrated, regular oscillations are composed of simple sine oscillations. In music, these are referred to as partial tones of a musical tone.[1] A single vibration of the above form consists of four partial tones:



Superposition of partial tones

While the first partial tone executes *one* oscillation, the second does *two*, the third *three*, and the fourth *four* vibrations. Accordingly, the partial tones vibrate at the basic frequency, double frequency, triple and quadruple frequency. The linear superposition of these oscillations results in a total vibration of the displayed form.

The illustration shows only a simple example. Musical tones normally consist of many more partial tones so that even the fifth, sixth, seventh etc. partial tones are contained in the sound. The sound spectrum of a tone contains a whole series of partial tones that are integer multiples of the basic vibration. Individual partial tones can vibrate more or less powerfully so that different waveforms are the result, which are perceived as specific timbres and acoustic colours. With the use of the Fourier analysis, it can be determined how strongly each single partial tone oscillates in the sound of a musical tone. This gives the characteristic sound spectrum, in which every partial oscillation is displayed with its frequency and volume.

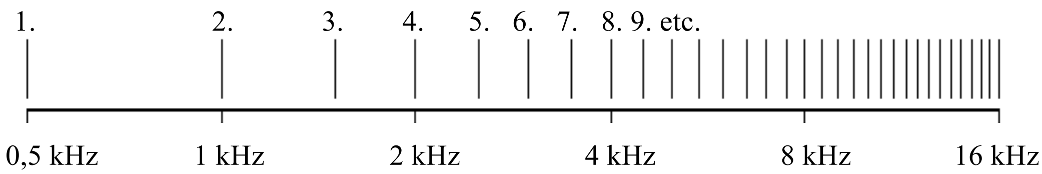


Sound spectrum of a violin tone of 440 Hz

[1] In a physical sense, partial tones are sine waves. In this regard, one speaks of "harmonic oscillations" or "harmonic motions". The physical attribute "harmonic" and the designation of partial tones as "harmonics" have nothing to do with harmony in an aesthetic sense.

The sound of a tone results from its inner composition. The fact that a tone always has a sound character is, on the one hand, quite natural because mechanical oscillations have a more or less pronounced sound spectrum, simply for physical reasons. On the other hand, the fact that musical tones are *rich* in partial tones is not a mere natural phenomenon, because it is the result of a sophisticated instrument making. The sonority of the tones is achieved by various techniques: by the positioning of the violin bow when stroking the strings, by resonance bodies, by octave couplers of an organ, etc. The musical sound is cultivated for its own sake; it is designed as an object of enjoyment.

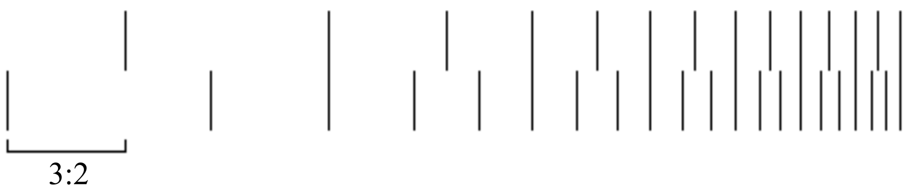
The quality of the tones by which they are predestined to harmonise can thus be summarised as follows: It is a matter of *sonorous* tones insofar as they are composed of a whole series of partial tones. Obviously, the harmony between the tones does not depend on their particular timbre and sound colour. The sonorous tone can therefore be represented schematically, by abstracting from the sonic intensity of the individual partial tones:



Schematic representation of the sound spectrum of a sonorous tone

In this illustration, the frequencies are scaled logarithmically. This representation has the advantage that equal frequency ratios appear as equal distances, as comes closest to the perspective of musicians who are familiar with this view due to the musical notation, the piano keyboard, etc.

By comparing the sound spectra of two tones that have the frequency ratio 3:2, one can see that every second partial tone of the higher tone oscillates at the same frequency as every third partial tone of the lower tone.



Consonance schema of the fifth

The two sonorous tones have thus coinciding partial tones. In the sounding-together of the tones, the oscillations of their common partial tones overlap to form a *single* partial oscillation in each case, in which even the Fourier analysis cannot indicate to which tone belongs which share in the sound intensity of a partial tone. Only the schematic separation of the tones can reveal the different origin of the partial tone in question.

Now it becomes apparent in what way both of the tones of a fifth harmonise: They match as sonorous tones, based on an accordance of their sound parts. This effect shall be called *basic consonance*.[1]

As said before, the harmony in the sounding-together of the sonorous tones is in any case independent of their specific timbre and acoustic colour. In the sound spectrum of tones, individual partial oscillations can therefore be completely absent without this diminishing the effect of the consonance. This applies, for example, to the clarinet, whose tones do not have even-numbered partials in its sound spectrum. The coincidence of partial tones also includes the case where a partial oscillation of a tone lies on a frequency that is a multiple of the basic frequency of another tone. That this other tone lacks a partial tone at the concerning position, is no reason for the ear to ignore the principle assignment of partial vibrations to a basic frequency. The detectable vibration has therefore partial tone character for both tones, even if it has its origin in only one of the tones. Basic consonance is therefore, strictly speaking, the harmony of a sound combination in which vibrations occur that have partial tone character for both of the sounding together tones alike.

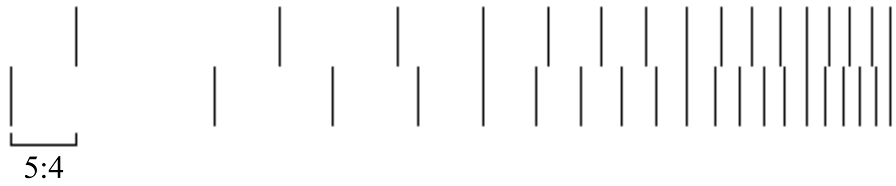
Experiments have shown that, after an adaption phase, test persons can consider relations between sinus tones – even if they are offered separately for each ear through headphones – to be consonances. Testers and test persons have the freedom to call acoustic phenomena consonances if they remind them of consonances, although individual sinus tones cannot harmonise at all. Such experiments do not disprove the concept of the consonance, but are at most evidence for the susceptibility of the perception for illusions, which can come about under the participation of other mental capacities like memory, imagination, mind, and interest.[2]

When schemata of different consonances are compared, one can see that the extent of the coincidence of partial tones can be expressed by the ratio of the total vibrations of the tones: In the case of the fifth, the frequency ratio 3:2 equals a harmony in which every *second* partial tone of the one tone coincides with every *third* one of the other tone. The frequency ratio of the major third (5:4) corre-

[1] Husmann formulated this realisation still most clearly: *“Since for the consonance the coincidence of common overtones would thus be responsible, the author has described his theory as the* coincidence theory *of the consonance ... It is the greater harmony of the, above the root tones rising, overtone structure of intervals with simple vibration ratios, which makes them appear consonant ...”* (Heinrich Husmann, *Einführung in die Musikwissenschaft*, Heidelberg 1958, pp. 134 f.)

[2] Husmann lets himself be theoretically confused by such experiments and concedes consonance in the relation of mere sine oscillations: *“Since at the binaural experiments, aside from the overtones, only the primary tones themselves are present, the assumption cannot be excluded that the consonance is also already founded in the primary tones alone ...”* (Ibid., p. 135). More about Husmann’s theory of the consonance in: Franz Sauter, *Die Musikwissenschaft in Forschung und Lehre. Kritik einer bürgerlichen Wissenschaft*, Norderstedt 2010, chapter 3.

sponds to a harmony in which every *fifth* partial tone of the one tone coincides with every *fourth* one of the other tone.

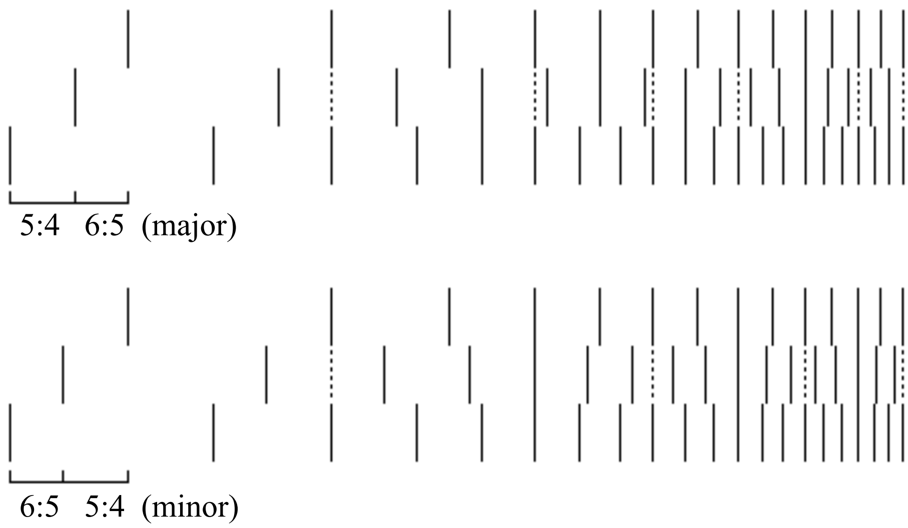


Consonance schema of a major third

The conspicuous numerical proportions, which are characteristic for harmonic sound combinations and which can be read off accordingly from the length ratios of vibrating strings or air columns, have therefore their reason in the sound architecture of the tones. The proportions that can be observed in the *external* tone relations are the necessary manifestation of the *inner* relations that the sonorous tones have to each other because of their internal structure. The essence of the harmony lies definitely not in the *numerical ratios*, but in the *going well together* of the tones that is based on their sound character.

The difference in the degree of the harmony, which can be seen at the consonance schemata, is intuitively felt when comparing tone ratios acoustically. Since the weaker or stronger harmony of the sounding-together tones is related to the proportion of their frequencies, which is easily determinable, the illusion arises as if the frequency ratios themselves have a harmonic character. The theoretical deepening of this confusion consequently ends up in number mysticism, which subscribes to the believe that the harmony that is inherent to the sounding-together of the tones is a property of the associated external numerical proportions.

The consonance of the fifth and thirds is summarized in the harmony of the major or minor triad:



Consonance schemata of a major and minor triad

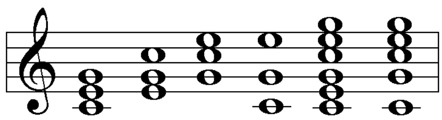
The mere summing-up of the consonances to a triad intensifies the effect of the basic consonance. The reason for this is that the combination of two consonances automatically includes a third consonance, whose harmony thus comes in addition ‘for free’. This is equally true for major and minor triads. However, the schematic representation of their consonance also shows a specific difference in the harmonic structure of the major and minor triads.

In case of the major triad, the coincidences of partial tones extend over more frequencies than in case of the minor triad. This is, for one thing, due to the fact that these coincidences are more widely dispersed in the major triad, whereas, in the minor triad, they are more concentrated on frequencies where all three tones have common partial oscillations. For another, there is more accordance in total in the major triad. Within the bandwidth illustrated above, the major triad has twelve times common partial oscillations, whereas the minor triad shows only ten such coincidences. This harmonic difference is the reason why musicians are able to intuitively distinguish major and minor triads on the basis of their sound. Also, the greater consonance of the major triad is noticeable when compared acoustically.

The consonant triad perfects the principle of the harmony that is contained in its components, the consonant dyads (fifth and thirds). It follows from this that the actual harmonic potency of the fifth and the thirds comes into effect in their determination as subordinate moments in the harmony of the consonant triads. Their harmonic role in tonal music is entirely in accordance with this: The fifth is a sound that is always defined by the sound relations of major and minor triads; it is divided in one way or another by thirds. Likewise, major and minor thirds are not autonomous harmonies, but are in each case components of either a major or a minor triad.

b) Compound Consonance

The musician distinguishes the basic form of a major and minor triad from the other forms in which individual tones are shifted to sound one or more octaves higher or lower or in which tones are supplemented by accordingly shifted tones. This distinction is determining for the practical handling of the harmonies in tonal music.

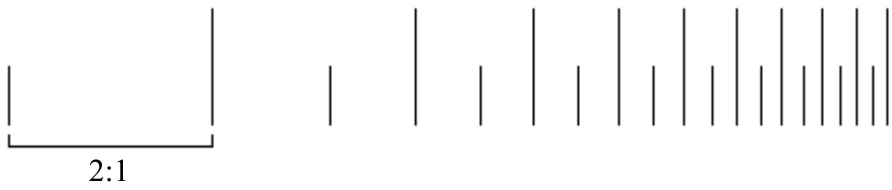


Basic form, inversions and extended forms of a C major triad

All of these sound shapes are named C major, even though they are quite different: Tones of varying numbers and positions sound together in it. On the other hand, it is not a matter of a random name equivalence, as is the case with persons, for example, who are all called Paul. Rather, it is a matter of an *appropriate* subsumption of sounds under a collective term. The use of the term ‘C major triad' in the above example shows that musicians consider an equal treatment of these sounds as reasonable. They therefore abstract from the differences of these sounds; and they do rightly so: They take into account that an actual commonality of these sounds do exist. But *what* is it that is being abstracted from and *what* is the commonality of all these sound shapes?

First of all, all of these chords sound very similar. In this respect, the commonality of the forms of a major or minor triad is their belonging to a family of sounds that differ only slightly when listening to. However, this similarity, which the musician intuitively grasps, lies on the level of the immediate appearance and does not justify a conceptual abstraction which can last against errors and misperceptions. In biology, for example, the external similarity of animal species is a starting point for a classification, but it does not necessarily reveal the animal's true nature. Accordingly, whales are, despite all appearances, not fish but mammals.

When musicians identify a major or minor triad independently of its particular form, they abstract from a difference that in fact only represents a harmonic nuance. It is a matter of that difference that makes itself felt when a tone within a sound combination is substituted or supplemented by another tone that lies one or more octaves higher or lower. Obviously, the sound relation of the octave does not create by itself any significant harmonic differences.

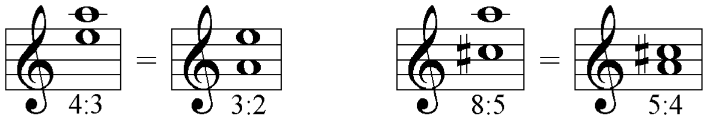


Consonance schema of the octave

Tones sounding together in an octave ratio appear as a dyad with an unsurpassable consonance because the complete sound of the upper tone coincides with half the sound of the lower tone. What presents itself as a similarity in the harmonic comparison of the different shapes of a triad is the same thing that appears as an extra strong harmony in the perception of the octave: the accordance of the sounds in regard to the frequencies of their partial tones. In realising major and minor triads and, accordingly, in the entire world of harmony that is based on major and minor triads, one thus *abstracts* from the harmony of the octave.

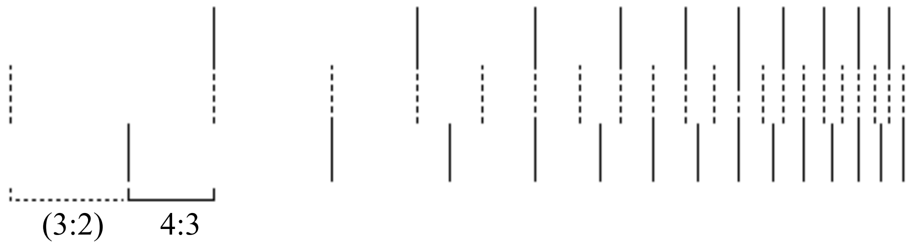
However, the neglect of the octave when determining major and minor triads is, in substance, not a disregard of the octave, but its consideration. The musical treatment of the octave fully corresponds to its harmonic property. That the octave causes only a negligible harmonic difference is only the flip side of a superlative of the consonance. The insignificance of the octave in a sphere, where it is about the harmonic identity of the sounds, is the form in which its absolutely dominant position in the harmonics of the tonal music comes into its own. The octave is therefore a substantial part of the consonant dyads which in their combination to major or minor triads perfect the principle of the consonance.

What applies to the major and minor triads as a whole, also applies to their components – the thirds and fifths. These, too, are to be understood abstractly and grasped in accordance with their harmonic essence against their specific manifestations:



Inversion of the fifth and of the major third

The fourth (4:3) is harmonically the same as the fifth (3:2), and the minor sixth (8:5) is the same as the major third (5:4), as Jean-Philippe Rameau already stated.[1] The reason is obviously the harmony of the octave, which determines the sound image of the fourth and sixth. The harmony of the in such a way modified shapes of the fifth and thirds shall be called *compound consonance*.



The fourth as a compound consonance

The dashed lines in the consonance schema of the fourth indicate an imaginary tone that forms an octave along with the upper tone of the fourth, and a fifth along with the lower tone. These lines illustrate the intermediate step in the harmonic constitution of a fourth. The harmony of the fourth is not reduced to the immediate coincidence of the partials in this sound combination, but unites the harmony of the octave and fifth:

$$4:3 = (2:3) \cdot (2:1)$$

The practical equation of the various forms of major and minor sounds thus allows a conclusion to be drawn on the specific capacity of the musical perception to intuitively break down sound relations into their simple components.

If one wanted to adequately express what the fourth is and what it represents for the perception, then one could write the frequency ratio in the following form: 2^2:3. But this is merely a mathematical illustration of the compound consonance at the level of its external frequency ratios. At this level, the consonance cannot be grasped anyway, and the formula

$$3:2 = 4:3$$

appears as a mathematical nonsense. Accordingly, the difference between the basic and compound consonance appears to be erased in the form of the frequen-

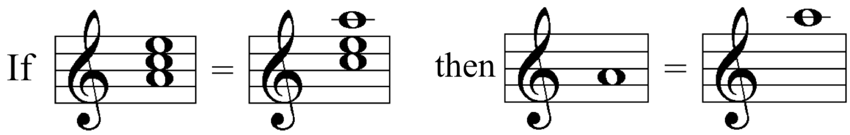
[1] Rameau thus calls the fourth the "shadow image" (l'ombre) of the fifth, following an expression of Descartes (Cf. Jean-Philippe Rameau, *Traité de l'harmonie réduite à ses principes naturels*, Paris 1722, p. 11).

cy ratios. Theorists who confuse the outward appearance of the harmonies with their essence locate the fourth harmonically between the fifth and the third because they are fascinated and dazzled by the mathematical series

2:1, 3:2, 4:3, 5:4, 6:5.

If the fifth and fourth are presented in such a series, then the difference in their kind of consonance is ignored as well as the commonality in their abstract harmonic essence. It is then not surprising that such a theory construction gets into trouble when, at first, it misinterprets the consonance as a relation of small numbers, to then speculate and wonder whether the relation 16:5 (a variant of the major third) can be a consonance or not.[1]

The abstract identity of the consonant sounds is finally reflected by the tones of which they consist:



The tones that sound together in a ratio of 2:1 are identical because and insofar it is abstracted from their insignificant contribution to the harmonic differentiation in all sound combinations. Thus, in addition to the concrete tone appears the tone in its abstract definition so that because of this double meaning of the term tone it can be said that the octave is a harmonic relation between identical tones. It is therefore quite consistent and appropriate if the respective tones are given the same name.[2]

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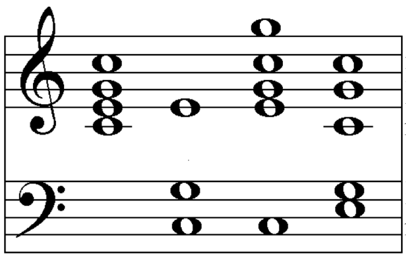
[1] An example of this speculation: *"The tones of consonant sounds stand in simple frequency proportions to each other. However, the consonance phenomenon can neither be clearly explained mathematically nor physically."* (Gerhard Kwiatkowski et al. (eds.), *Meyers kleines Lexikon Musik*, Mannheim et al. 1986, p.186). The theorists have blocked their path to a *musical* explanation of the consonance by their preference for *extra-musical* patterns of interpretation.

[2] Jean-Phillipe Rameau already noticed that the same naming of different tones reflects an objective harmonic fact. With such terms as "inversion", "root tone" etc., he coined the generally accepted terminology used today, which takes this discovery into account. However, he did not realise the abstraction on the basis of which it makes sense to speak of identical tones in the case of the octave. Likewise, he overlooked the underlying harmony (the fitting-together) of the tones because he did not doubt the (up to the present day) popular idea of a harmony that is inherent to the numerical proportions. In the ratio of 2:1, he therefore sought to find a property that would make the identity of the tones plausible: *"La proportion du tout à sa moitié, ou de la moitié au tout est si naturelle, qu' elle se conçoit d' abord; ce qui doit nous prévenir en faveur de l'Octave, dont la raison est comme 1. à 2. l'Unité est le principe des nombres, & 2. en est le premier, se trouvant un grand rapport entre ces deux Epithetes,* Principe & Premier*, dont l'application est tres-juste. Aussi dans la pratique, l'Octave n'est distinguée que sous le nom de réplique;…"* (Rameau, *Traité de l'harmonie réduite à ses principes naturels*, p. 6)

The musician usually imagines the matter the other way round: The identity of two harmonies seems logical to him insofar as the harmonies consist of the same tones. And that these tones are equal is not familiar to him as a harmonic abstraction, but by the names under which the tones were introduced to him. That works very well in practice. Why racking one's brain over the absurdity that language rules are supposed to have an effect on the identity of harmonies?

The abstract definition of the tones is also reflected in the designation of the tones of major and minor triads: Root tone, third, and fifth are, regardless of their respective octave position, the only components of these sounds, which exactly therefore are consonant *triads* in the abstract sense, and not only in the basic form analysed at the beginning, but in all other forms as well. The lowest tone of a chord is therefore not the same as its root tone. This also applies to the harmonic components of the triads: The fourth has its root tone on top, that is, it is the fifth turned upside-down.



Root tone, third and fifth in C major triads

c) Theories About Major and Minor Triads

Gioseffo Zarlino was the first one to express the opinion that major and minor triads are the basic shapes of the harmonics. He opposed the then usual treatment of the thirds as "imperfect" sound combinations. This treatment was based on the Pythagorean tuning, which deduces all tone relations from fifths so that the frequency ratio of the major third does not have the numeric value of

$$5:4 \quad = \ 1.25$$

but

$$81:64 \ = \ 1.265625$$

In order to prove the harmonic character of major and minor triads, however, he lacked the necessary fundamentals, namely the knowledge about the inner structure of the tones. Actually, he should have realised that he was not capable of explaining the harmonic principle of the consonance. Nevertheless, he realised that consonant tone relations can be demonstrated by dividing a vibrating string by 2, 3, 4, 5, and 6. As a matter of fact, this results in the oscillation ratios of the octave, fifth, fourth, and thirds. This induced Zarlino to assume that the reason for the correlation between the consonance and the observed numerical proportions lies in a specialty of the number six. To that end he applied two traditional thought patterns of the mystical numerology:

First, he claims to know a mathematical quality of the number six, which he calls ‘perfect’. He, thereby, creatively complements the presentation of the known types of numbers like prime numbers etc.: *“Perfect numbers are those which can be added up from their parts...”*[1] In order to make the assumed perfection clear, he says that the number six has *“a certain measure whereby nothing is too much and nothing too little.”*[2]

Second, he calls attention to the role of the number six in many important contexts. This starts in the Holy Scriptures right off with the story of creation and is then soon broken off for good reasons: *“It would take a long time to recount all the things in detail that come to a completion with the number six.”*[3]

The effort to prove specific tone relations to be necessary manifestations of the consonance never gets to the essence of the consonance in this way because it does not thematise *its* property, but instead the property of a number. As an explanation of major and minor triads, it is offered that they be attested by laws that are supposed to prevail *regardless* of any music. This is an interpretation within the framework of a worldview.

200 years later, when it was known that the musical tones consist of partial tones, nothing more would have stood in the way of exploring the consonance. In fact, the major and minor triads were associated with partial tones, but merely as a continuation of the endeavour to substantiate the aesthetic quality of these triads with outside existing phenomena. The inner structure of the tones serves from now on in speculative music theory as a model, as a pattern of nature, to which harmony is imitated. The appeal of this model lies in its character as an undoubted and experimentally verifiable natural phenomenon. This is supposed to guarantee a harmonic quality separately from the examined harmonies.

Very common is the interpretation of the major triad as a replication of the fourth, fifth, and sixth partial tones. In the case of the minor triad, one discovers either a direct model that is given in the relation of the 10th, 12th, and 15th partial tones; or one believes that only dyads are “naturally” prefigured in the overtone series, namely the thirds, from which major and minor triads are then “artificially” put together; or one imagines a reverse “undertone series”, which, however, is not detectable in nature, but instead fits well into the imagination of a *“truth of nature of the dual harmony principle ...”*.[4] The speculative musicology can draw on an inexhaustible pluralism of such theories, which classify themselves within a factional dispute between monists and dualists and which are

[1] Gioseffo Zarlino, *Theorie des Tonsystems. Das erste und zweite Buch der Istituzioni harmoniche (1573)*, trans. Michael Fend, Frankfurt am Main 1989, p. 79.

[2] Ibid., p. 84.

[3] Ibid., p. 85.

[4] Hugo Riemann, *Handbuch der Harmonielehre*, Leipzig 1918, p. 214.

supported in this classification by an academic reporting system that is rather striving for a rubrication of theories than for a scientific explanation.[1]

The contrasts of the theories on major and minor triads are of a purely ideological nature. The opponents agree in the ideological starting point of the debate: What they do not notice about harmony is that it is the work of aesthetically interested subjects and their materialism eager for sound enjoyment; but instead they assume the effective force of a higher principle given in nature, according to which the whole world is *constructed* in the truest sense of the word. The most popular arguments are therefore confessions to the respectively own world view, as can be seen from the following polemic against dualism:

"Everything earthly is looking for a support point ... However, if the nature only builds upwards and lets grow, but never also 'symmetrically-contrarily' downward, then the 'will of nature', presented for proof, is non-existent, and the proof is untenable."[2]

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[1] An example for this stereotypical thinking: *"Major and minor triads have therefore sprung up from one root ... This system is called 'monism' ... Riemann's major and minor triads spring up from different roots ... This is the system of dualism."* (Hermann Grabner, *Allgemeine Musiklehre*, Kassel 1974, p. 155)

[2] Josef Achtélik, *Der Naturklang als Wurzel aller Harmonien*, pt. 2, Leipzig 1922, p. 104.